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Extension of the compensated distortion method to the critical heat flux modelling in rectangular inclined channel

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Abstract

Parameters of critical heat flux modelling in inclined rectangular channels for high-pressure convective two-phase flows have been derived using the compensated distortion method of Ahmad [Int. J. Heat Mass Transfer 16 (1973) 641]. Fréon R12 was used as the modelling fluid. A specific expression for the modelling parameter has been obtained by introducing a new dimensionless parameter that accounts for the effect of inclination on critical heat flux. This parameter is built by balancing the inertial force acting on a particle against the viscous effect corrected by a transverse term of gravity. In the same time, a dimensionless correlation of critical heat flux is proposed, which describes available data with an r.m.s. error of 13.1%.

Keywords: Critical heat flux; Dimensionless parameter; Correlation; Inclination; Compensated distortion; Rectangular channel

1. Introduction

Boiling is a very efficient mode of heat transfer encountered in many industrial fields like nuclear power plants, vapor generators or heat exchangers. Unfortunately, under some operating conditions, this mechanism is limited by the appearance of the boiling crisis. Due to the destructive character of this phenomenon, it is therefore the utmost importance to be able to predict as accurately as possible the operating conditions leading to its appearance. Nevertheless, despite the fact that such a phenomenon has been studied for several decades, the mechanisms that can explain the boiling crisis are not yet well understood, especially for the case of the *departure from nucleate boiling*. Due to this lack of reliable modelling, an empirical approach—using the working fluid and the full-scale geometries—is often used. Unfortunately, such experiments are often costly (for steam-water systems, due to the high-pressure or high temperature levels). The modelling technique [1,6] is an alternative method that allows alleviating the working conditions. It consists on replacing the working fluid (e.g. *water*) by a modelling fluid with a *lower latent heat of vaporization* so that the power and the temperature levels are considerably reduced. A model from which quantitative data about the behaviour of the prototype can be obtained consequently replaces the original physical system e.g. the prototype. The similarity laws relating these two systems can be obtained via a dimensional analysis of the studied problem. This approach leads to the derivation of scaling laws that allows the resolution of the model operating conditions as a function of the prototype ones. But if the fluid-to-fluid modelling technique has so far been applied to critical heat flux in vertical or horizontal channels for various geometries (e.g. circular, annular or rod bundles), very few studies concern the case of inclined rectangular channels. The outline of this paper is first to show that the method originally proposed by Ahmad can be extended to the present study's conditions, then in a second step to provide and to test a set of scaling laws specific to such a configuration.

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Nomenclature

A	characteristic scale of length [m]
d	characteristic scale of length [m]
D	diameter [m]
g	gravity [m/s^2]
G	mass flux [kg/m^2 s]
f_i	inertial force [N]
f_v	viscous force [N]
f_g	buoyancy force [N]
H_{inlet}	inlet enthalpy [kJ/kg]
H_{Lsat}	saturated liquid enthalpy [kJ/kg]
h_{LG}	latent vapor heat [kJ/kg]
L	length [m]
L_{ch}	heated length [m]
n_i, p_i	empirical constants [–]
r^2	linear regression coefficient [–]
r.m.s.	root mean square [–]
S_C	heating area [m^2]
S_P	flow area [m^2]
V	characteristic scale of velocity [m/s]
x_i	dimensional variable [–]
X_S	thermodynamic quality at dryout [–]

Greek symbols

δ	distortion factor [–]
μ	characteristic scale of viscosity [Ns/m^2]
μ_G	gas viscosity [Ns/m^2]
μ_L	liquid viscosity [Ns/m^2]
φ	heat flux [kW/m^2]
ρ	characteristic scale of density [kg/m^3]
$\overline{\rho}$	average density [kg/m^3]
ρ_G	gas density [kg/m^3]
ρ_L	liquid density [kg/m^3]
σ	surface tension [N/m]
σ_v	standard deviation [–]
θ	inclination to the vertical [°]

Dimensionless parameters

Bo	boiling number [–]
Bo_m	modified boiling number [–]
F_r	Froude number (Eq. (6)) [–]
F_r^*	Froude number (Eq. (10)) [–]
Γ_1	Öhnesorge number [–]
Γ_2	viscosity ratio [–]
Γ_3	bond number [–]
K	Kutateladze number [–]
Λ_2	sub-cooling number [–]
Λ_3	density number [–]
Λ_4	geometric number [–]
Ω_i	adjustable dimensionless number [–]
Π_i	dimensionless number [–]
ϖ_i	non-adjustable number [–]
ψ_{CHF}	modelling parameter (Eq. (5)) [–]
ψ_H	modelling parameter (Eq. (7)) [–]
Re_m	modified Reynolds number [–]
ζ_{CHF}	modelling parameter (Eq. (17)) [–]

2. Literature survey

Studies related the boiling crisis scaling laws are numerous. One can quote the works of Barnett [2], Stevens and Kirby [8], Bouré [3], Ahmad [1] and Merilo [6]. Such works can be classified in two families:

- the empirical method e.g. Barnett [2] and Stevens and Kirby [8],
- the semi-empirical methods e.g. Bouré [3], Ahmad [1] and Merilo [6].

But if the first methods are still used nowadays because of their simplicity, their empirical nature makes their extrapolation beyond their validity range very hazardous. Semi-empirical methods have a more general character, and despite the fact that they also need some experimental adjustment, their range of validity are often larger than the empirical methods because they are based on a more physical analysis. Among the semi-empirical methods, the one initially developed by Ahmad [1] then recovered by Merilo [6] is particularly interesting. Indeed, when a small number of dimensionless groups properly describes the behaviour of a physical system, it is possible to get an exact model by ensuring that all the dimensionless groups are identical for both model and prototype. But for more complex systems, it quickly becomes difficult to ensure such a requirement. In this situation, we have an incomplete simulation and the model appears to be distorted. Ahmad proposed a method for compensating these distortions.

Ahmad [1] studied the critical heat flux in vertical tubes for various geometries. From the general method described in Appendix A], he finally proposed five dimensionless groups e.g.:

- the boiling number:

$$Bo \triangleq \frac{\varphi}{G h_{LG}} \tag{1}$$

- the sub-cooling number:

$$\Lambda_2 \triangleq \frac{H_{inlet} - H_{Lsat}}{h_{LG}} \tag{2}$$

- the density number:

$$\Lambda_3 \triangleq \frac{\rho_L}{\rho_G} \tag{3}$$

- the geometric number:

$$\Lambda_4 \triangleq \frac{L}{D} \tag{4}$$

- the modelling parameter:

$$\psi_{\mathrm{CHF}} \triangleq \Lambda_5 \triangleq Re \times \Gamma_1^{n_1} \times \Gamma_2^{n_2} = \left(\frac{GD}{\mu_L}\right)\left(\frac{\mu_L^2}{\sigma\rho_L D}\right)^{4/3}\left(\frac{\mu_L}{\mu_G}\right)^{-1/5} \tag{5}$$

The equality of the dimensionless parameters Λ_2 to Λ_5 between the model and the prototype implies the equality of the parameter Bo for both model and prototype. The values of the exponents n_1 and n_2 are obtained (see Appendix B) for the following operating conditions:

- geometry: circular tube,
- fluid: water, Fréon R12,
- pressure: 7 MPa.

Figs. 1 and 2 show the results obtained by Ahmad when applying his modelling criteria to different experimental critical heat flux data. We can see that in all the cases, the modelling and prototype CHF data approximately fall on the same curve. Such results indicate both the success of the modelling technique and its generality because it has been tested for very various experimental working conditions (different fluids, geometry or thermal hydraulics operating conditions).

Using Ahmad's method, Merilo [6] proposed scaling laws for the critical heat flux modelling in a horizontal tube. In such a geometric configuration, Merilo showed that the influence of the gravity cannot be neglected especially for low velocities (Fig. 3). Mérilo suggested to include in the expression of Ahmad's modelling parameter ψ_{CHF}, a specific term accounting for the influence of gravity.

Thus, several attempts have been made using different dimensionless parameters like the Froude number:

$$F_r = \frac{G}{\rho_L\sqrt{gD}} \tag{6}$$

or the Kutateladze number:

$$K = \frac{G}{\rho_L}\left[\frac{\rho_L^2}{\sigma g(\rho_L - \rho_G)}\right]^{1/4} \tag{7}$$

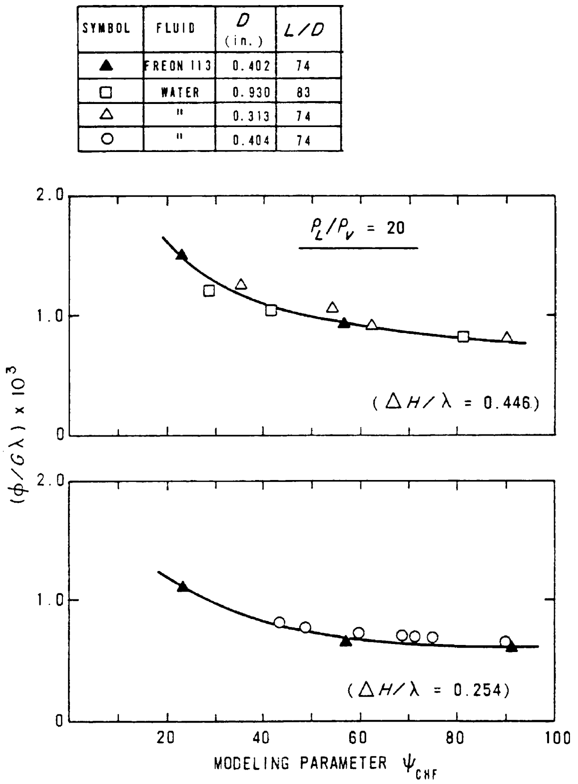

Fig. 1. Boiling number Bo vs. modelling parameter ψ_{CHF} $P = 7$ MPa—vertical tube (from [1]).

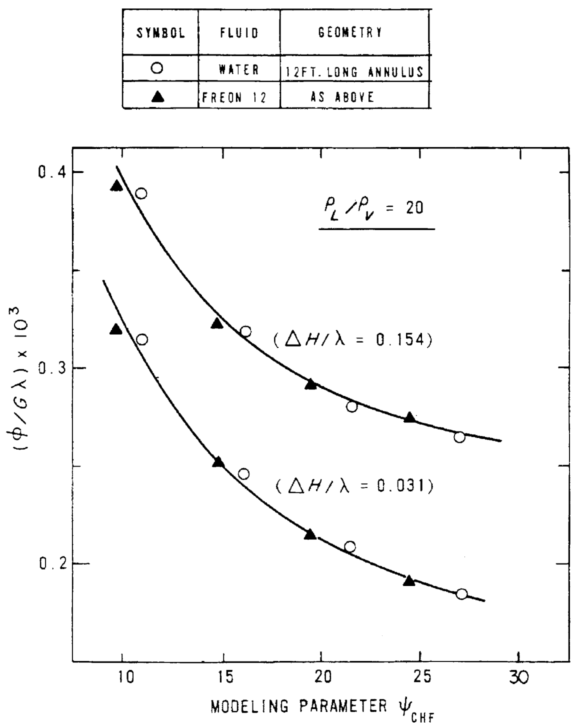

Fig. 2. Boiling number Bo vs. modelling parameter ψ_{CHF} $P = 7$ MPa—vertical annular tube (from [1]).

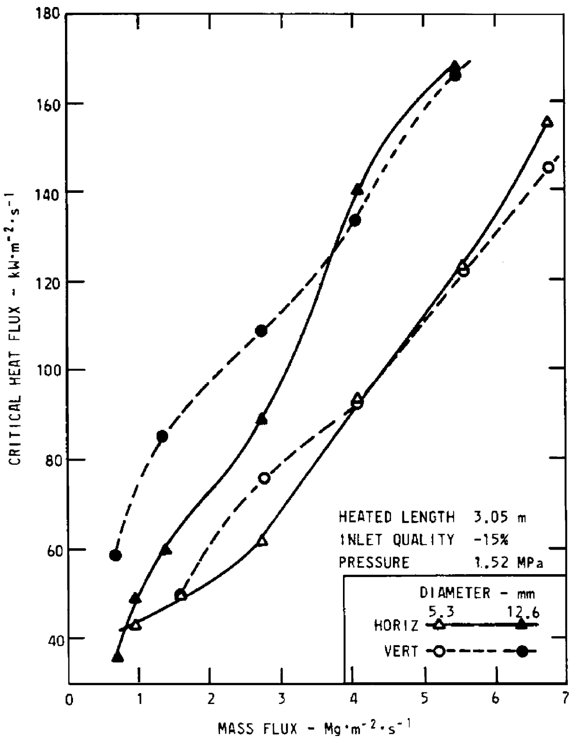

Fig. 3. Critical heat flux vs. mass flux (from [7]).

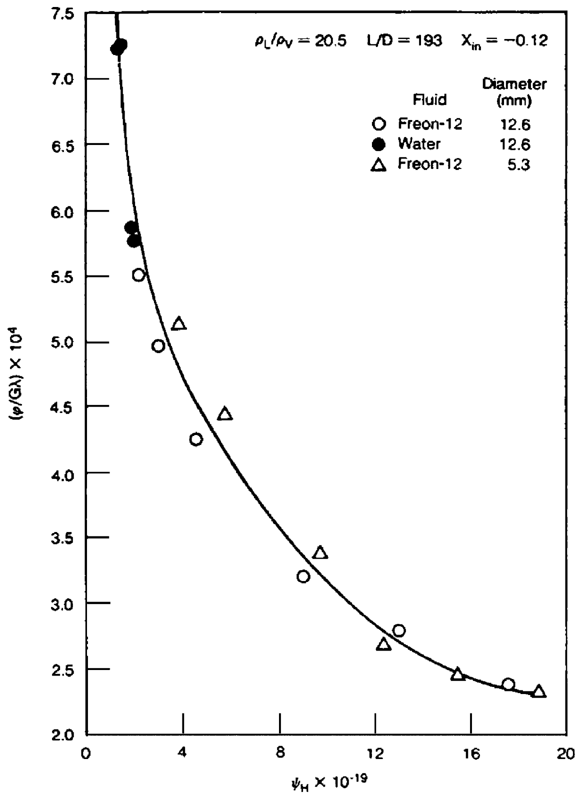

Fig. 4. Boiling number Bo vs. modelling parameter ψ_{H} $P = 7$ MPa—horizontal tube (from [6]).

Merilo finally decided to choose the Bond number because its inclusion gives a lower r.m.s. error in the final correlation:

$$Bo = \frac{(\rho_{\mathrm{L}} - \rho_{\mathrm{G}})gD^2}{\sigma} \tag{8}$$

then, he proposes the following expression for the modelling parameter:

$$\psi_{\mathrm{H}} \triangleq Re \times \Gamma_1^{p_1} \times \Gamma_2^{p_2} \times Bo^{p_3} \tag{9}$$

where Re, Γ_1, Γ_2 are defined by Eq. (5) and Bo by Eq. (8).

As the values of the non-dimensional parameters Γ_2 and Bo cannot be varied independently, the values of the exponents p_1, p_2, p_3 have been calculated from experimental boiling crisis data (e.g. Table 1) by a correlative approach (see Appendix B).

Figs. 3–5 confirm the validity of the modelling parameter expression ψ_{H} used by Merilo.

It should however be noted that the experimental data used to correlate the modelling parameter expression are also in the same time used to establish its validity. On that account, the results obtained by Merilo appear to be foreseeable, so that the conclusions stated by Merilo should be examined carefully.

However, and despite the last remark, due to the results obtained by both Ahmad [1] and Merilo [6], the compensated distortion method's appears to be a very general and interesting approach in order to develop a set of scaling laws.

Unfortunately, none of these works concern the case of high-pressure convective flow in an inclined rectangular channel. More generally, a literature survey shows that very few studies are related to so specific configuration. One can cite the work of Cumo and Naviglio [4]

Table 1
Range of experimental parameters tested by Merilo [6]

Fluid	Tube diameter D [mm]	Ratio L/D	$\rho_{\mathrm{L}}/\rho_{\mathrm{G}}$	Mass flux [kg/m^2 s]	Inlet quality X_{e} [%]
Fréon R12	12.6	193–387	13–20.5	700–5400	−35 to 0
Water	12.6	193–387	13–20.5	1000–5700	−30 to 0
Water	19.1	112–160	20.5	700–1400	−29 to 7
Fréon R12	5.3	193–571	13–20.5	1600–8100	−35 to 0

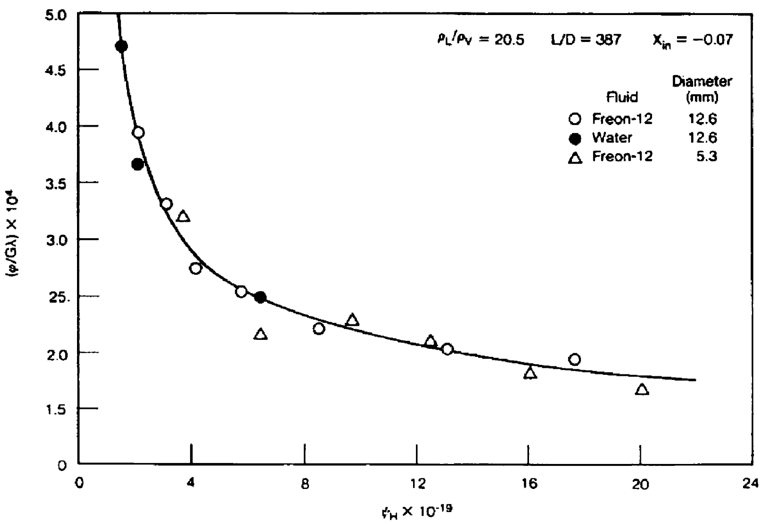

Fig. 5. Boiling number *Bo* vs. modelling parameter ψ_H $P = 7$ MPa—horizontal tube (from [6]).

and the one of Dimmick [5] who both studied the CHF in operating conditions close to the ones of interest.

Cumo and Naviglio [4] studied the inclination influence on the boiling crisis for high-pressure convective flow in a cylindrical tube. They used Fréon R12 as coolant. The pressure laid between 1 and 3 MPa (which is equivalent to 7–18 MPa in water according to Ahmad's scaling criteria), the mass flux between 600 and 2000 kg/m^2 s, the inclination to the vertical between 0° and 60°, and a positive exit quality.

They showed that all the experimental data can be correlated by the following non-dimensional parameter:

$$F_r^* \triangleq \frac{G\cos(\theta)}{\rho_L\sqrt{gD\frac{(\rho_L-\rho_G)}{\rho_L}}} \tag{10}$$

This parameter can be interpreted as the ratio between the vertical component of the fluid inlet velocity and a velocity proportional to the one of a rising bubble in a stagnant liquid. Cumo and Naviglio also showed that if the value of this parameter is greater than 6, phase stratification caused by the inclination of the channel becomes very significant.

Dimmick [5] studied the effect of orientation on CHF in a 3-rod bundle cooled by Fréon R12. His aim consisted in deriving a non-dimensional CHF correlation that can apply whatever the inclination of the channel. In his experiments, the pressure laid between 0.8 and 1.5 MPa (which is equivalent to 5–10 MPa in water), the mass flux between 500 and 4000 kg/m^2 s, the exit quality greater than 0. Three values of the inclination have been tested: $\theta = 0°$ (vertical position), $\theta = 11°$, 90° (horizontal position). He assumed that in the case of vertical tube, a non-dimensional correlation of critical heat flux can be written as below:

$$Bo \triangleq \Im\left[Re, \left(1+\frac{H_{\text{inlet}}-H_{\text{Lsat}}}{h_{\text{LG}}}\right), \left(\frac{\rho_L}{\rho_G}-1\right)\right] \tag{11}$$

To account for the effect of orientation, Dimmick proposes to modify the expression of the Reynolds number to include a transverse gravity term (Fig. 6).

Let us consider a two-phase flow constituted of gas bubble and liquid flow. Let d and V respectively denote

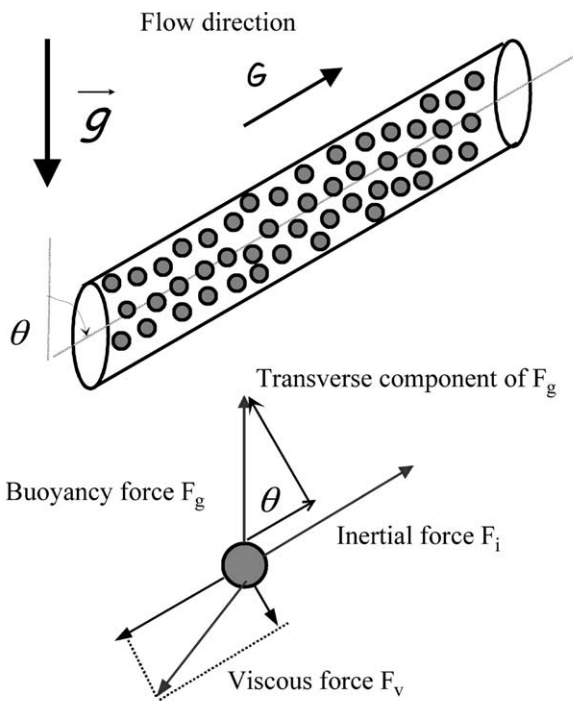

Fig. 6. Schematic diagram of the forces acting on a particle in an inclined channel.

the characteristic scales for the particle size and for its velocity.

Balancing inertial force against the difference between the viscous and the transverse component of the buoyancy force gives:

$$Re_m \sim \frac{f_i}{f_v - f_g} \sim \frac{\rho V^2 d^2}{\mu V d - g\,\Delta\rho d^3 \sin\theta} \tag{12}$$

We can notice that if $\theta = 0°$, Eq. (12) resume to the classical bubble Reynolds number as shown below:

$$\frac{f_i}{f_v} \sim \frac{\rho V d}{\mu} \tag{13}$$

If it is quite easy to define characteristic scales for the velocity, the phase density or the viscosity, it is much more difficult for the particle size characteristic scale because, in a two-phase flow (especially with phase change), this size is always changing and can change from several orders of magnitude. That is the reason why, classically, in the Reynolds number, the characteristic size if often chosen as being *the channel diameter D*. Such a choice means a distortion on length's characteristic size. To compensate this distortion, Dimmick proposed *to introduce in the expression of the buoyancy force*, a new characteristic size designed A instead of d, where A is unknown and has therefore to be used as a weighting factor.

Eq. (12) can be rewritten:

$$Re_m = \frac{\rho D^2 V^2}{\mu D V - g A^3 \Delta\rho \sin\theta} \tag{14}$$

Expressing the velocity in terms of mass flux G and assuming homogeneous flow so that we can define:

$$\begin{cases} \overline{\rho} = \rho_L - X_S(\rho_L - \rho_G) \\ G = \overline{\rho} V \end{cases} \tag{15}$$

Eq. (14) can be rewritten:

$$Re_m = \frac{DG}{\mu - \frac{gA^3 \sin\theta}{DG}\left[\rho_L(\rho_L - \rho_G) - X_S(\rho_L - \rho_G)^2\right]} \tag{16}$$

Combining Eqs. (11) and (16) leads to the following relation:

$$Bo = \Upsilon\left[Re_m, \left(1 + \frac{H_{inlet} - H_{Lsat}}{h_{LG}}\right), \left(\frac{\rho_L}{\rho_G} - 1\right)\right] \tag{17}$$

Dimmick chose a power law form for the function Υ. The value of the weighting factor A is determined by a statistical technique of optimisation which leads for Dimmick's experiments to $A = 0.330$ m. The r.m.s. of the obtained correlation is 3.4%.

Dimmick did not give more physical explanation about the value of the parameter A.

3. Construction of a similitude

As specified before, our problem can be distinguished by two specificities:

- the channel geometry (rectangular),
- its inclination from 0° (vertical) to 45°.

Owing to its general character, it was decided to use Ahmad's modelling technique. So we tried to modify the scaling laws originally proposed by Ahmad [1] in order to account for both particularities specified above.

If the sub-cooling number and the density number appear to be relevant to our study, on the other hand it does not seem to be the case for the boiling number, the geometric number or the modelling parameter. indeed, the formulation of these non-dimensional parameters is intimately linked to the studied problem. Thus, it seems to be necessary to modify these non-dimensional numbers to account for our problem specificities.

To account for the inclination, it was decided to change Ahmad's modelling parameter by replacing in Eq. (5), the classical Reynolds number by its modified expression (Eq. (16)), where A will be a statistical adjustment parameter. So a new modelling parameter expression is defined:

$$\zeta_{CHF} \triangleq Re_m \times \left(\frac{\mu_L^2}{\sigma\rho_L D_H}\right)^{m_1} \times \left(\frac{\mu_L}{\mu_G}\right)^{m_2} \tag{18}$$

which can be rewritten:

$$\zeta_{CHF} \triangleq \left(\frac{D_H G}{\mu_{Lsat} - \frac{gA^3 \sin\theta}{D_H G}\left[\rho_L(\rho_L - \rho_G) - X_S(\rho_L - \rho_G)^2\right]}\right) \times \left(\frac{\mu_L^2}{\sigma\rho_L D_H}\right)^{m_1} \times \left(\frac{\mu_L}{\mu_G}\right)^{m_2} \tag{19}$$

It can be noticed that if the inclination is equal to zero, the expression (19) is strictly equivalent to the expression (5).

With regard to the problem of the geometry, most of the studies related to boiling crisis scaling laws [1–3,8], outlined that both model and prototype should have very close geometries and often propose as non-dimensional criterion the ratio aspect L/D.

Two reasons can essentially warrant the interest for this parameter:

- First, for a hydrodynamic point of view: this number allows to characterize the entrance effects on the development of a two-phase flow.
- Secondly for a thermal point of view: If we consider a simple energy balance for a tube—with a diameter D and a heated length L_{ch}—, we have:

$$\frac{\varphi}{G h_{LG}} \frac{S_C}{S_P} = \frac{H_{inlet} - H_{Lsat}}{h_{LG}} + X_S \tag{20}$$

where S_C and S_P are respectively the heated and the flow area.

For a cylindrical tube, Eq. (20) is:

$$\frac{\varphi}{Gh_{LG}}\frac{L}{D} = \frac{H_{inlet} - H_{Lsat}}{h_{LG}} + X_S \tag{21}$$

So we can see that if the boiling number, the sub-cooling number and the geometric number are identical between the model and the prototype, it will also be the case for the thermodynamic mass quality X_S.

In our problem, the geometry of both prototype and model are not totally adjustable. In particular, even if they are very close (same gap and same axial heated length), their dimensions do not ensure the conservation of the ratio between the heated and the flow area. Nevertheless, in order to ensure the equality of the thermodynamic mass quality between the model and the prototype, we propose to replace the classical boiling number and the ratio aspect by a single non-dimensional parameter built as a combination of the previous ones:

$$Bo_m \triangleq \frac{\varphi}{Gh_{LG}}\frac{S_C}{S_P} \tag{22}$$

However, it is still necessary to ensure that the geometry of both model and prototype are quite close (in terms of geometric aspect and in terms of dimensions).

4. Summary

Being inspired by Ahmad's works, and after adjusting them to the present problem, we finally proposed a boiling crisis similitude based on the respect between both model and prototype of the four following non-dimensional parameters:

$$\left.\frac{\rho_L}{\rho_G}\right]_{prototype} = \left.\frac{\rho_L}{\rho_G}\right]_{model} \tag{23}$$

which sets the equivalent pressure between both systems,

$$\left.\frac{H_{inlet} - H_{Lsat}}{h_{LG}}\right]_{prototype} = \left.\frac{H_{inlet} - H_{Lsat}}{h_{LG}}\right]_{model} \tag{24}$$

which sets the equivalent initial sub-cooling between both systems,

$$\zeta_{CHF}]_{prototype} = \zeta_{CHF}]_{model} \tag{25}$$

which determines the equivalent mass flux between both systems,

$$Bo_m]_{prototype} = Bo_m]_{model} \tag{26}$$

which finally sets the equivalent heat flux between the model and the prototype.

5. Determination of constants m_1, m_2, A

The values of the constants m_1, m_2 and A are determined from experimental data. For simplicity reasons, it was decided to use the correlative approach as in [6]. According to Ahmad and Merilo's works, a power law was therefore chosen for the CHF correlation.

$$Bo_m = aRe_m^b \left(\frac{\mu_L^2}{\sigma \rho_L D_H}\right)^c \left(\frac{\mu_L}{\mu_G}\right)^d \left(1 + \frac{H_{inlet} - H_{Lsat}}{h_{LG}}\right)^e \times \left(\frac{\rho_L}{\rho_G} - 1\right)^f \tag{27}$$

As the modified Reynolds number depends on the variable A, an iterative scheme (Fig. 7) has been used.

For every value of the parameter A, the constants a, b, c, d, e, f are computed with a multi-linear regression method over a part of the data bank (Table 2). The final value of A is the one that leads to the best fitting between the correlation and the experimental data. The accuracy of the fitting can be characterized by two parameters which are:

- The linear regression coefficient r^2 that should be as close as possible of 1.
- The standard deviation σ_v that should be as small as possible.

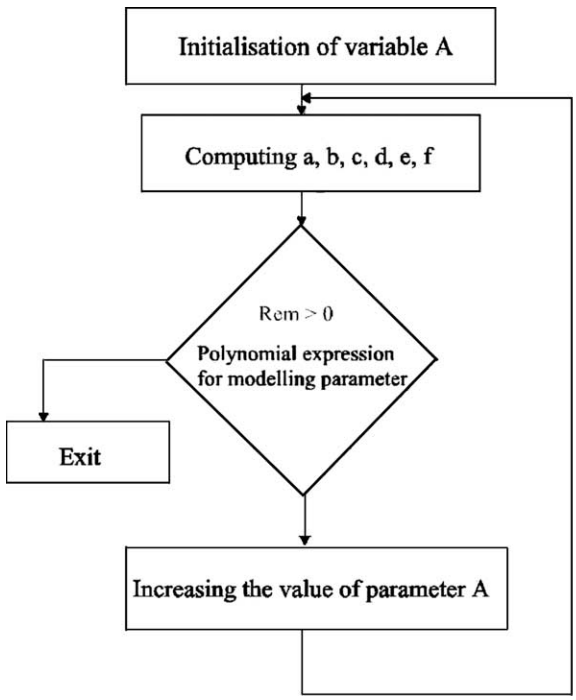

Fig. 7. Algorithm for computing the value of parameter A.

Table 2
Range of experimental parameters used for the critical heat flux correlation

Fluid	Dh [mm]	L/Dh	Inclination	Mass flux [kg/m^2 s]	Inlet quality [%]
Water	5–10	100–200	0°/15°/30°/45°	200–2000	−60 to 0

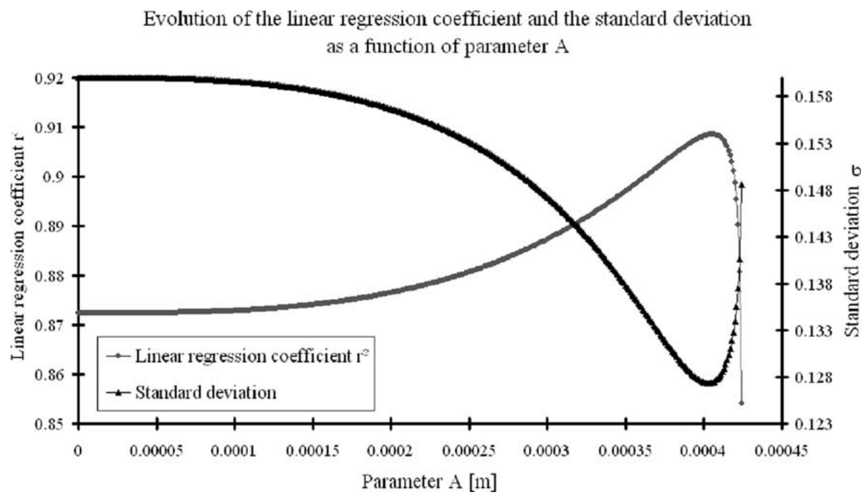

Fig. 8. Evolution of the coefficient r^2 and the coefficient σ_v as a function of A.

The ranges of the experimental data used to build the correlation are shown in Table 2. More particularly, it can be seen that there were no reference to Fréon data for the development of the empirical expression of the modelling parameter. The remaining data of the data bank will be used for the similitude validation.

Fig. 8 shows the evolution of the coefficients r^2 and σ_v as a function of A. It can seen that there is an optimal value of A ($A = 0.404$ mm) that leads to a maximal value for r^2 ($r^2 = 0.91$) and in the same time a minimal value for σ_v ($\sigma_v = 12.7\%$). The correlation can be then written:

$$Bo_m = 3905 Re_m^{-0.47}\left(\frac{\mu_L^2}{\sigma\rho_L D_H}\right)^{0.22}\left(\frac{\mu_L}{\mu_G}\right)^{-0.83} \times\left(1+\frac{H_{inlet}-H_{Lsat}}{h_{LG}}\right)^{-0.32}\left(\frac{\rho_L}{\rho_G}-1\right)^{0.51} \tag{28}$$

From Eqs. (19) and (28), the modelling parameter ζ_{CHF} can easily be deduced to be:

$$\zeta_{CHF} = Re_m \times \left(\frac{\mu_L^2}{\sigma\rho_L D_H}\right)^{-0.476} \times \left(\frac{\mu_L}{\mu_G}\right)^{1.765} \tag{29}$$

This correlation has been applied to the former experimental data by considering each inclination separately. It can seen in Table 3 that the standard deviation is quite similar for each inclination. Such a result confirm that the inclination influence is well accounted for by the modelling parameter ζ_{CHF} (Eq. (19)).

Table 3
Standard deviation of the correlation for each inclination

Inclination	Standard deviation σ_v
$\theta = 0°$	13.8%
$\theta = 15°$	11.9%
$\theta = 30°$	13.1%
$\theta = 45°$	13.4%

6. Comparison of the modelling technique with experimental data

The success of the proposed modelling technique can be ensured by plotting the modified boiling number Bo_m as a function of the modelling parameter ζ_{CHF}, the remaining dimensionless parameters being held constant. If the density and the sub-cooling numbers are the same for model and prototype, both sets of critical heat flux data drawn in the plane Bo_m vs. ζ_{CHF} should fall on the same curve.

Typical results are shown in Figs. 9–11. It can seen that the data for different experiments (Table 4) fall approximately on the same curve. This indicates the success of modelling technique and ensures the validity of the proposed scaling criteria.

These results are partially confirmed by the application of the above CHF correlation (Eq. (28)) to the Fréon data (Table 4). We can see on Fig. 12 that there is a good agreement between the experimental data and the correlation. The r.m.s. error is 12.8%.

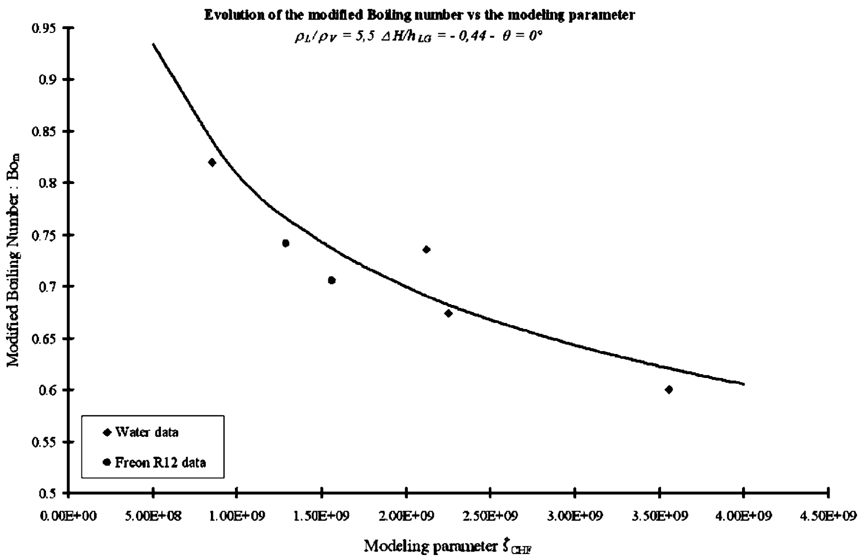

Fig. 9. Evolution of the boiling number vs. the modelling parameter—$\theta = 0°$.

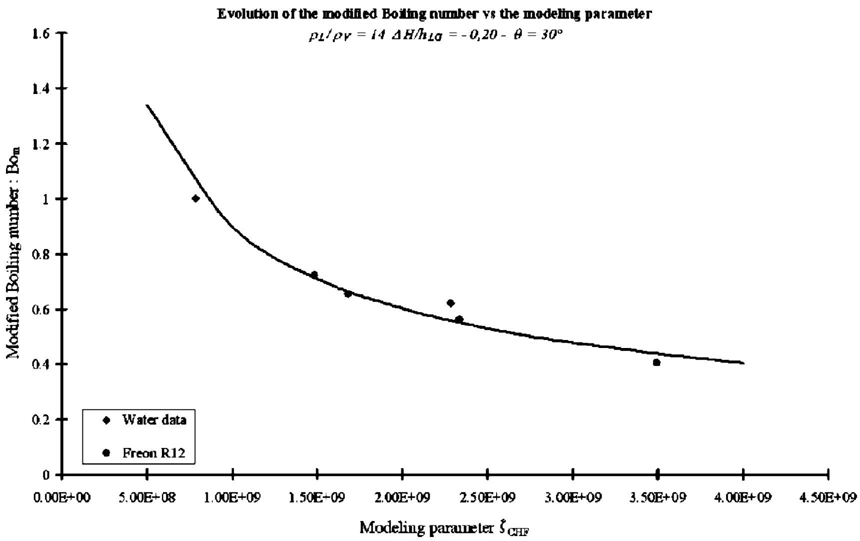

Fig. 10. Evolution of the boiling number vs. the modelling parameter—$\theta = 30°$.

7. Conclusion

The concept of compensated distortion modelling technique, originally developed by Ahmad [1] for vertical two-phase flows, then adjusted by Merilo [6] for horizontal two-phase flows has been extended to the case of inclined channels, for different inclinations (from 0° to 45°).

To account for channel inclination, a new dimensionless parameter Re_m has been introduced in the set of scaling laws proposed by Ahmad. This number has been obtained by balancing the inertial force against the viscous force corrected by a transverse term of gravity.

In the same time, a dimensionless critical heat flux correlation has been derived from water data in rectangular inclined channel. It describes the available data with an r.m.s. error of 13.1%. Tested against Fréon R12 data, it shows a good agreement with the experimental data (r.m.s. of 12.8%).

Such results confirm the very powerful character of the modelling technique.

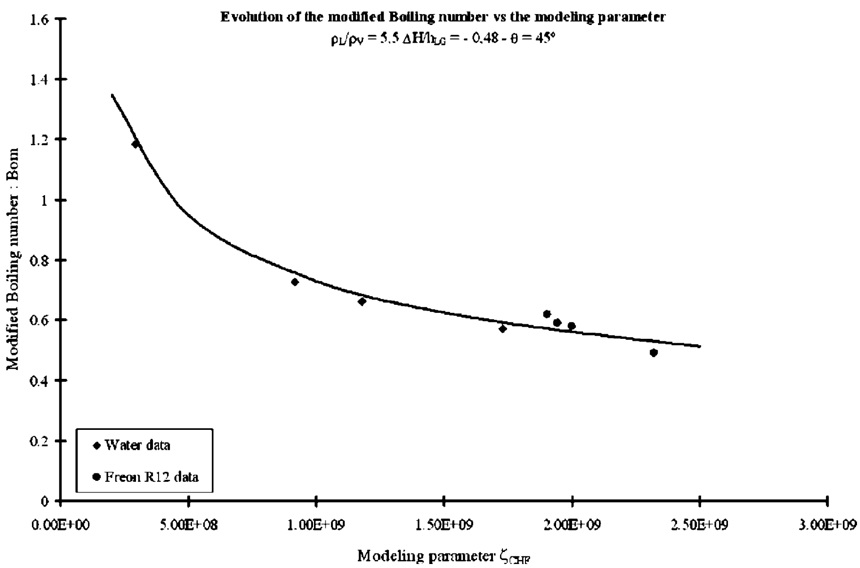

Fig. 11. Evolution of the boiling number vs. the modelling parameter—$\theta = 45°$.

Table 4
Range of experimental parameters used for similitude validation

Fluid	Dh [mm]	L/Dh	Inclination	Mass flux [kg/m^2 s]	Inlet quality [%]
Fréon R12	5–10	100–200	0°/30°/45°	200–2000	−30 to 0

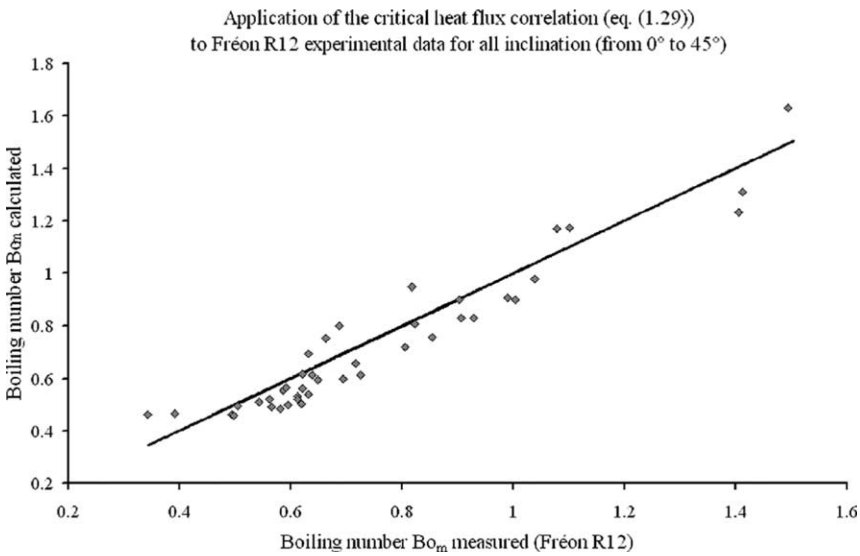

Fig. 12. Application of the critical heat flux correlation to Fréon R12 experimental data.

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Appendix A

Let us consider a physical phenomenon that can be described by the following relation:

$$\varepsilon\left([x_i]_{i\in[1;n]}\right) = 0 \tag{A.1}$$

where $[x_i]_{i\in[1;n]}$ are the dimensional variables that are supposed to be of first importance for the studied problem. By using the Buckingham's theorem [9], this can be rewritten in a dimensionless form as follows:

$$\upsilon\left([\Pi_i]_{i\in[1;p]}\right) = 0 \tag{A.2}$$

where $[\Pi_i]_{i\in[1;p]}$ are dimensionless parameters deduced from the variables $[x_i]_{i\in[1;n]}$.

If we suppose that:

- Π_1 points out the dimensionless number holding the *dependent variable* (e.g. the flux density if we study the critical heat flux),
- $[\Omega_i]_{i\in[1;r]}$ point out the dimensionless numbers that are adjustable (by acting on the regulation parameters of the experiment),
- $[\varpi_i]_{i\in[1;s]}$ point out the others dimensionless numbers (that are not adjustable),

the expression (A.2) can be rewritten:

$$\Pi_1 = \omega\left([\Omega_i]_{i\in[1;r]}, [\varpi_j]_{j\in[1;s]}\right) \tag{A.3}$$

The similarity between the prototype and the model sets to have:

$$\Pi_1]_{\text{prototype}} = \Pi_1]_{\text{model}} \tag{A.4}$$

Such an equality can be satisfied on the condition that:

$$\forall i \in [1;r], \quad \Omega_i]_{\text{model}} = \Omega_i]_{\text{prototype}} \tag{A.5}$$

$$\forall j \in [1;s], \quad \varpi_j]_{\text{model}} = \varpi_j]_{\text{prototype}} \tag{A.6}$$

If Eq. (A.5) can be easily satisfied (because all the parameters Ω_i are adjustable on both model and prototype), it is much more difficult for the parameters $[\varpi_j]_{j\in[1;s]}$.

Thus Eq. (A.4) can be rewritten:

$$\begin{cases} \Pi_1]_{\text{prototype}} = \delta\Pi_1]_{\text{model}} \\ \delta \neq 1 \end{cases} \tag{A.7}$$

The compensated distortion method consists on choosing a dimensionless adjustable parameter among $[\Omega_i]_{i\in[1;r]}$, e.g. Ω_1, then to define an arbitrary function η so that Eq. (A.3) could be rewritten:

$$\Pi_1 = \vartheta\left([\Omega_i]_{i\neq 1}, \eta\left(\Omega_1, [\varpi_j]_{j\in[1;s]}\right)\right) \tag{A.8}$$

Then, Eq. (A.4) will be satisfied provided that the following equality are respected:

$$\begin{cases} \forall i \in [2;r], \quad \Omega_i]_{\text{prototype}} = \Omega_i\Big]_{\text{model}} \\ \forall j \in [1;s], \quad \eta(\Omega_i, \varpi_j)\big]_{\text{prototype}} = \eta(\Omega_1, \varpi_j)\Big]_{\text{model}} \end{cases} \tag{A.9}$$

The function η is called the distortion parameter and Ω_1 the compensation parameter. The form of the function η is empirically determined.

Appendix B

The method used by Ahmad [1] to compute the values of the constants n_1 and n_2 is graphical.

It consists on plotting the evolution of the boiling number Bo as a function of the Reynolds number Λ_1 (for n_1) for two different values of the Öhnesorge number Γ_1 (the values of Λ_2, Λ_3, Λ_4, Γ_2 are imposed), then to determine the value of n_1 so that both curves collapses in a single one.

The method is symmetric for computing n_2. One have to plot boiling number Bo as a function of $\Lambda_1 \times \Gamma_1^{n_1}$ for two different values of the viscosity ratio Γ_2.

There are two conditions for the use of this method:

- First, it is necessary to have data related to two different geometries. Indeed, for an imposed value of Λ_2, Λ_3, Λ_4, Γ_2, Γ_1 can only be changed by acting on the geometry of the system.
- Secondly, Γ_1 and Γ_2 should be independent, so that one can be adjusted without changing the value of the other.

The method proposed by Merilo [6] is purely correlative. On that account, this method can be always be employed, especially when Ahmad's approach is inapplicable (e.g. when the dimensionless numbers used in the modelling parameter are not independent). It consists on establishing a general correlation for computing the dimensionless dependent variable Π_1 as a function of all the remaining dimensionless parameters. For Merilo's case, it reverts to look for a correlation as follows:

$$\Pi_1 = a \times \Lambda_1^b \times \Lambda_2^c \times \Lambda_3^d \times \Lambda_4^e \times \Gamma_1^f \times \Gamma_2^g \times \Gamma_3^h \times \tag{B.1}$$

As the expression of the modelling parameter is:

$$\psi_{\text{H}} = \Lambda_1 \times \Gamma_1^{p_1} \times \Gamma_2^{p_2} \times \Gamma_3^{p_3} \times \tag{B.2}$$

it is possible to determine the values of the constants p_1, p_2 and p_3 by identification between Eqs. (B.1) and (B.2):

$$\begin{cases} p_1 = \frac{f}{b} \\ p_2 = \frac{g}{b} \\ p_3 = \frac{h}{b} \end{cases} \tag{B.3}$$

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